

**Subject group and course title: Group 5 – Mathematics – Applications and Interpretations.**

**Course purpose:**

The aims of all DP mathematics courses are to enable students to:

1. develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
2. develop an understanding of the concepts, principles and nature of mathematics
3. communicate mathematics clearly, concisely and confidently in a variety of contexts
4. develop logical and creative thinking, and patience and persistence in problem solving to instil confidence in using mathematics
5. employ and refine their powers of abstraction and generalization
6. take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities
7. appreciate how developments in technology and mathematics influence each other
8. appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics
9. appreciate the universality of mathematics and its multicultural, international and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course
11. develop the ability to reflect critically upon their own work and the work of others
12. independently and collaboratively extend their understanding of mathematics.

Problem solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP mathematics course, students will be expected to demonstrate the following:

1. **Knowledge and understanding:** Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.
2. **Problem solving:** Recall, select and use their knowledge of mathematical skills, results and models in both abstract and real-world contexts to solve problems.
3. **Communication and interpretation:** Transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation; use appropriate notation and terminology.
4. **Technology:** Use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems.
5. **Reasoning:** Construct mathematical arguments through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions.
6. **Inquiry approaches:** Investigate unfamiliar situations, both abstract and from the real world, involving organizing and analyzing information, making conjectures, drawing conclusions, and testing their validity.

## Topics covered (in order taught during the two years)

### - Topic 1 – Number and Algebra

- a) Operations with numbers in scientific form notation.
- b) Arithmetic and geometric sequences and series and their applications. Sum of the first  $n$  terms of a sequence.
- c) Compound interest - annual depreciation.
- d) Laws of exponents with integer and rational exponents.
- e) Introduction to logarithms with base 10 and  $e$ . Numerical evaluation using technology.
- f) Approximation, decimal places, significant figures. Upper and lower bound, rounding.
- g) Percentage errors and estimation.
- h) Ammortization and annuities using technology
- i) Solving polynomial equations and systems of linear equations in up to 3 variables, using technology.

### Topic 2—Functions

- a) Different forms of the equation of a straight line. Gradient; intercepts.
- b) Concept of a function, domain, range and graph, notation. Functions as a mathematical model.
- c) Inverse functions as a reflection in the line  $y = x$  and the notation  $f^{-1}(x)$ .
- d) Using technology to graph functions including their sums and differences.
- e) Determine key features of graphs. (maximum, minimum, intercepts, symmetry, vertex, roots of equations, vertical and horizontal asymptotes using graphic technology)
- f) Points of intersection of two curves, lines using technology.
- g) Modelling with the following functions: Linear models  $f(x) = mx + c$ , quadratic models  $f(x) = mx^2 + c$ , exponential growth and decay models, direct/inverse variation, cubic models, sinusoidal models.
- h) Modelling skills, test and reflect upon a model they developed based on the previous mentioned functions. Reading, interpreting and making predictions based on models.

### - Topic 3— Geometry and trigonometry

- a) The distance between two points in three - dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.
- b) Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. Sine rule, cosine rule, area of triangle using  $\sin$ .
- c) Applications of right and non-right angled trigonometry, including Pythagoras's theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.
- d) The circle; length of an arc; area of a sector.
- e) Equations of perpendicular bisectors.
- f) Voronoi diagrams.

### - Topic 4— Statistics and probability

- a) Concepts of population, sample, random sample, discrete and continuous data. Reliability of data sources and bias in sampling. Interpretation of outliers. Sampling techniques and their effectiveness.
- b) Presentation of data (discrete and continuous): frequency distributions (tables).

- c) Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR).
- d) Production and understanding of box and whisker diagrams.
- e) Measures of central tendency (mean, median and mode). Estimation of mean from grouped data. Modal class. Measures of dispersion (interquartile range, standard deviation and variance). Quartiles of discrete data.
- f) Linear correlation of bivariate data. Pearson's product-moment correlation coefficient,  $r$
- g) Scatter diagrams; lines of best fit, by eye, passing through the mean point.
- h) Equation of the regression line of  $y$  on  $x$ . Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters,  $a$  and  $b$ , in a linear regression  $y = ax + b$ .
- i) Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space ( $U$ ) and event. The probability of an event. The complementary events  $A$  and  $A'$  (not  $A$ ).
- j) Venn diagrams, Combined events, Conditional probability, Independent events.
- k) Concept of discrete random variables and their probability distributions. Expected value (mean), for discrete data. Applications.
- l) Binomial distribution. Mean and variance of the binomial distribution.
- m) The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation. Normal probability calculations, inverse normal calculations.
- n) Spearman's rank correlation coefficient. Awareness of the appropriateness and limitations of Pearson's product moment correlation and Spearman's rank correlation coefficient, and the effect of the outliers on each.
- o) Formulation of null and alternative hypotheses, significance levels,  $p$ -values. The  $\chi^2$  test for independence, the  $\chi^2$  goodness of fit test. The  $t$  - test. Using one-tailed and two-tailed tests.

#### - Topic 5 – Calculus

- a) Limits. Derivatives. Derivatives as gradients and as rate of change. Increasing and decreasing functions.
- b) Derivatives of various basic functions. Tangents and normals at a given point and their equations.
- c) Integrations as anti-differentiation of polynomial functions.
- d) Definite integral using technology. Area of a region enclosed by a graph.
- e) Values of  $x$  where the gradient of a curve is zero. Solution of  $f'(x) = 0$ .
- f) Local maximum and minimum points. Optimization problems in context.
- g) Approximating areas using the trapezoidal rule.

#### Assessment model

**Knowledge and understanding:** Students should be able to demonstrate their ability of solving exercises and use the appropriate method according to the problem – exercise assigned.

**Problem solving:** Students work on their ability to use their mathematical skills to solve abstract and real-world problems.

**Communication and interpretation:** Transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation; use appropriate notation and terminology.

**Technology:** Use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems.

**Reasoning:** Construct mathematical arguments through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions.

**Inquiry approaches:** Investigate unfamiliar situations, both abstract and from the real world, involving organizing and analyzing information, making conjectures, drawing conclusions, and testing their validity.

Throughout the course, the teaching will aim to:

- ✓ encourage inquiry and investigation.
- ✓ focus on the understanding of clear concepts and real -life connections of the content.
- ✓ focus on effective teamwork and collaboration and the ability of communication.
- ✓ differentiate the level of difficulty to meet learners needs.
- ✓ assess students' progress with regular feedback as well as with a final exam.

#### **Assessment tools**

During class we use PowerPoint presentations, weekly assignments, group or individual homework. After the coverage of each chapter a formative assessment (quizz) is conducted. In addition to this, exit tickets are given to students for checking their understanding. Semester exams and end of year exams are used as summative assessment.

#### **How are key concepts served (methodology)?**

Critical Thinking, transfer of knowledge to real-life context, links with other subjects and the use of technology are essential in DP Mathematics. According to the topic various concepts are served and examined as listed below:

**Topic 1:** Generalization, representation, modelling, equivalence, quantity, approximation.

**Topic 2:** Representation, relationships, space, change, modelling.

**Topic 3:** Generalization, space, relationships, systems, representations.

**Topic 4:** Quantity, validity, approximation, modelling, relationships, patterns.

**Topic 5:** Change, patterns, relationships, approximation, generalization, space.

#### **How does the course foster international mindedness?**

Mathematics is considered to be an international language with mathematicians being able to communicate worldwide with it. The connection of History and Mathematics and the way mathematics have been developed are promoted in Maths class. Mathematics is known as the language of sciences and technology. Opportunities are given to students to promote mathematics into a range of local and global issues and ideas.

#### **How are IB Learner Profile attributes promoted?**

The IB Learner profile has a vital role for the Diploma Programme. An IB Student would not be an IB

learner if he has not achieved various aspects of the learner profile. During teaching various activities are designed in order to serve this scope. Activities require students to be investigative, inquirers, promote communication as they mostly require students to work in teams and encourage knowledge and reflection.

### **How does the course meet student needs via ATL?**

The five approaches to learning (developing thinking skills, social skills, communication skills, self-management skills and research skills) along with the six approaches to teaching (teaching that is inquiry-based, conceptually-focused, contextualized, collaborative, differentiated and informed by assessment) encompass the key values and principles that underpin IB pedagogy.

### **Describe connections with CAS**

Creativity, activity, service (CAS) is at the heart of the Diploma Programme. All courses have to be aligned and promote CAS activities. Students are engaged in various CAS activities that evolve maths as followed:

1. They create a mathematics scavenger hunt for younger students.
2. They assist other students in Math classes, usually younger students.
3. They create surveys and databases in order to resolve problems of the school community or area.

### **Describe connections with TOK**

Theory of knowledge (TOK) is a course that is fundamentally about critical thinking and inquiry, the two attributes that are mostly required in mathematical knowledge. During the course Math students are given questions and topics that are expected to discuss during TOK classes and investigate. They are encouraged to investigate further and finally a TOK and Maths combined course is designed to promote the TOK and Maths connection.

### **Recommended resources**

[www.ibo.org](http://www.ibo.org)

Khan Academy

TedEd

### **Instructor's name**

**Fouseki Panagiota, M.Sc**