IB DIPLOMA COURSE OUTLINE 2021 – 22

Subject group and course title: Group 5 – Mathematics – Analysis and Approaches

Course purpose:

The aims of all DP mathematics courses are to enable students to:

1. develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
2. develop an understanding of the concepts, principles and nature of mathematics
3. communicate mathematics clearly, concisely and confidently in a variety of contexts
4. develop logical and creative thinking, and patience and persistence in problem solving to instil confidence in using mathematics
5. employ and refine their powers of abstraction and generalization
6. take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities
7. appreciate how developments in technology and mathematics influence each other
8. appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics
9. appreciate the universality of mathematics and its multicultural, international and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular "area of knowledge" in the TOK course
11. develop the ability to reflect critically upon their own work and the work of others
12. independently and collaboratively extend their understanding of mathematics.

Problem solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP mathematics course, students will be expected to demonstrate the following:

1. **Knowledge and understanding**: Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.
2. **Problem solving:** Recall, select and use their knowledge of mathematical skills, results and models in both abstract and real-world contexts to solve problems.
3. **Communication and interpretation**: Transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation; use appropriate notation and terminology.
4. **Technology:** Use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems.
5. **Reasoning:** Construct mathematical arguments through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions.
6. **Inquiry approaches:** Investigate unfamiliar situations, both abstract and from the real world, involving organizing and analyzing information, making conjectures, drawing conclusions, and testing their validity.

Topics covered (in order taught during the two years)

- **Topic 1 – Number and Algebra**

a) Operations with numbers in scientific form notation.
b) Arithmetic and geometric sequences and series and their applications. Sum of infinite convergent geometric sequences.
c) Compound interest - annual depreciation.
d) Laws of exponents with integer and rational exponents.
e) Introduction to logarithms with base 10 and e. Numerical evaluation using technology.
f) Laws of logarithms and change of base.
g) Solving exponential equations including using logarithms.
h) Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side (LHS to RHS) proof. The symbols and notation for equality and identity
i) The binomial theorem.
j) Use of Pascal's triangle and ${}^{n}_{r}C$.

HL students are also taught:

a) The extension of the binomial theorem to fractional and negative indices.
b) Counting principles, including permutations and combinations.
c) Partial fractions.
d) Complex numbers: (number i, Cartesian form,the terms real part, imaginary part, conjugate, modulus and argument)
e) The complex plane. Modulus–argument (polar) form. Euler form: $z = re^{i\theta}$. Sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation
f) Complex conjugate roots of quadratic and polynomial equations with real coefficients.
g) De Moivre's theorem and its extension to rational exponents. Powers and roots of complex numbers.
h) Proofs (by mathematical induction, contradiction, use of counterexample)
i) Solutions of systems of linear equations (a maximum of three equations in three unknowns)

Topic 2—Functions

a) Different forms of the equation of a straight line. Gradient; intercepts.
b) Concept of a function, domain, range and graph, notation. Functions as a mathematical model.
c) Using technology to graph functions including their sums and differences.
d) Determine key features of graphs. (maximum, minimum, intercepts, symmetry, vertex, roots of equations, vertical and horizontal asymptotes using graphic technology)
e) Points of intersection of two curves, lines.
f) Composite functions. Inverse function. Identity function.
g) Quadratic function and graphs. Axis of symmetry. Solution of quadratic equations and inequalities. Quadratic formula. The discriminant.
h) The reciprocal function, graph, rational functions and graphs, equations of vertical and horizontal equations.
i) Exponential and Logarithmic functions and their graphs.
j) Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.
k) Composite transformations.
l) Transformations of graphs. Translations. Reflections in both axes.

HL students are also taught:

a) Polynomial functions, their graphs and equations; Factor and remainder theorems.
b) Rational functionsq
c) Odd and even functions. Finding the inverse function, including domain restriction. Self – inverse functions.
d) Solutions of g(x) ≥ f(x), both graphically and analytically.
e) The graphs of the functions, $y = |f(x)|$ and $y = f(|x|)$, $y = \frac{1}{f(x)}$, $y = f(ax + b)$, $y = [f(x)]^2$.

- **Topic 3— Geometry and trigonometry**

a) The distance between two points in three dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.
b) Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. Sine rule, cosine rule, area of triangle using sin
c) Applications of right and non-right angled trigonometry, including Pythagoras's theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.
d) The circle: radian measure of angles; length of an arc; area of a sector.
e) Definition of $cos\theta$, $sin\theta$ in terms of the unit circle. Definition of $tan\theta$.Exact values of various trigonometric ratios and their multiples.
f) The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$. Double angle identities for sine and cosine. The relationship between trigonometric ratios.
g) The circular functions $sinx$, $cosx$, and $tanx$; amplitude, their periodic nature, and their graphs Composite functions of the form $f(x) = asin(b(x + c)) + d$. Transformations, real – life context.
h) Solving trigonometric equations in a finite interval, both graphically and analytically.
i) Equations leading to quadratic equations in sinx, cosx or tanx.

HL students are also taught:

a) Definition of the reciprocal trigonometric ratios $sec\theta$, $cosec\theta$ and $cot\theta$. Pythagorean identities. The inverse functions $f(x) = arcsinx, f(x) = arccosx, f(x) = arctanx$; their domains and ranges; their graphs.
b) Compound angle identities. Double angle identity for tan.
c) Relationships between trigonometric functions and the symmetry properties of their graphs.
d) vectors (Concept of a vector; position vectors; displacement vectors. Representation of vectors using directed line segments, Base vectors i, j, k. Sum and difference of two vectors, the zero vector, vector 0.
e) The definition of the scalar product of two vectors. The angle between two vectors. Perpendicular vectors; parallel vectors, position vectors, magnitude, displacement vectors.
f) Vector equation of a line in two and three dimensions. The angle between two lines. Simple applications to kinematics.
g) Coincident, parallel, intersecting and skew lines, distinguishing between these cases. Points of intersection.
h) The definition of the vector product of two vectors.
i) Properties of the vector product.
j) Geometric interpretation of $|v \times w|$.

k) Vector equations of a plane
l) Intersections of a line with a plane; two planes; three planes. Angle between: a line and a plane; two planes.

- **Topic 4— Statistics and probability**

a) Concepts of population, sample, random sample, discrete and continuous data. Reliability of data sources and bias in sampling. Interpretation of outliers. Sampling techniques and their effectiveness.
b) Presentation of data (discrete and continuous): frequency distributions (tables).
c) Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR).
d) Production and understanding of box and whisker diagrams.
e) Measures of central tendency (mean, median and mode). Estimation of mean from grouped data. Modal class. Measures of dispersion (interquartile range, standard deviation and variance). Quartiles of discrete data.
f) Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, r
g) Scatter diagrams; lines of best fit, by eye, passing through the mean point.
h) Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression y = ax + b.
i) Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. The probability of an event. The complementary events A and A' (not A).
j) Venn diagrams, Combined events, Conditional probability, Independent events.
k) Concept of discrete random variables and their probability distributions. Expected value (mean), for discrete data. Applications.
l) Binomial distribution. Mean and variance of the binomial distribution.
m) The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation
n) Equation of the regression line of x on y. Use of the equation for prediction purposes.
o) Conditional probabilities.
p) Standardization of normal variables (z- values). Inverse normal calculations where mean and standard deviation are unknown.

HL students:

a) Bayes' theorem for a maximum of three events.
b) Variance of a discrete random variable. Continuous random variables.
c) Mean, variance and standard deviation of both discrete and continuous random variables.
d) Linear transformations for X.

- **Topic 5 – Calculus**

a) Limits. Derivatives. Derivatives as gradients and as rate of change. Increasing and decreasing functions.
b) Derivatives of various basic functions. Tangents and normals at a given point and their equations.
c) Integrations. As anti-differentiation of polynomial functions.
d) Definite integral using technology. Area of a region enclosed by a graph.
e) Differentiation of various functions. Chain rule for composite functions.
f) Second derivative and the graphical behavior of functions.
g) Local maximum and minimum points. Testing for maximum and minimum. Optimization. Examples of optimization may include profit, area and volume. Points of inflexion with zero and

integrals, including non-zero gradients.

h) Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled.
i) Indefinite integral.
j) Integration by inspection (reverse chain rule) or by substitution.
k) Definite analytical approach. Areas of a region enclosed by a curve $y = f(x)$ and the x-axis, where f(x) can be positive or negative, without the use of technology. Areas between curves.

High Level students should also study:

a) Informal understanding of continuity and differentiability of a function at a point.
b) The evaluation of limits using l'Hôpital's rule or the Maclaurin series.
c) Implicit differentiation. Related rates of change. Optimization problems.
d) Derivatives of $tanx, secx, cosecx, cotx, a^x, logax, arcsinx, arccosx, arctanx$.
e) Use of partial fractions to rearrange the integrand.
f) Integration by substitution and parts. Repeated integration by parts.
g) Area of the region enclosed by a curve and the y axis in a given interval. Volumes of revolution about the x-axis or y-axis.
h) First order differential equations.
i) Solution of y' + P(x)y = Q(x), using the integrating factor.
j) Maclaurin series to obtain expansions $for\ e^x, sinx, cosx, ln(1 + x), (1 + x)\,p, p \in \mathbb{Q}$.
k) Maclaurin series developed from differential equations.

Assessment model

Knowledge and understanding: Students should be able to demonstrate their ability of solving exercises and use the appropriate method according to the problem – exercise assigned.

Problem solving: Students work on their ability to use their mathematical skills in both abstract and real-world contexts to solve problems.

Communication and interpretation: Transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation; use appropriate notation and terminology.

Technology: Use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems.

Reasoning: Construct mathematical arguments through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions.

Inquiry approaches: Investigate unfamiliar situations, both abstract and from the real world, involving organizing and analyzing information, making conjectures, drawing conclusions, and testing their validity.

Throughout the course, the teaching will aim to:
- ✓ encourage inquiry and investigation.
- ✓ focus on the understanding of clear concepts and real -life connections of the content.

- ✓ focus on effective teamwork and collaboration and the ability of communication.
- ✓ differentiate the level of difficulty to meet learners needs.
- ✓ assess students' progress with regular feedback as well as with a final exam.

Assessment tools

During class we use PowerPoint presentations, weekly assignments, group or individual homework. After the coverage of each chapter a formative assessment (quizz) is conducted. In addition to this, exit tickets are given to students for checking their understanding. Semester exams and end of year exams are used as summative assessment.

How are key concepts served (methodology)?

Critical Thinking, transfer of knowledge to real-life context, links with other subjects and the use of technology are essential in DP Mathematics. According to the topic various concepts are served and examined as listed below:

Topic 1: Generalization, representation, modelling, equivalence, patterns, quantity, validity, systems.

Topic 2: Representation, relationships, space, quantity, equivalence, systems, patterns.

Topic 3: Generalization, space, relationships, equivalence, representation, quantity, modelling.

Topic 4: Quantity, validity, approximation, generalization, change, systems.

Topic 5: Change, patterns, relationships, approximation, generalization, space, modelling, systems, quantity.

How does the course foster international mindedness?

Mathematics is considered to be an international language with mathematicians being able to communicate worldwide with it. The connection of History and Mathematics and the way mathematics have been developed are promoted in Maths class. Mathematics is known as the language of sciences and technology. Opportunities are given to students to promote mathematics into a range of local and global issues and ideas.

How are IB Learner Profile attributes promoted?

The IB Learner profile has a vital role for the Diploma Programme. An IB Student would not be an IB learner if he has not achieved various aspects of the learner profile. During teaching various activities are designed in order to serve this scope. Activities require students to be investigative, inquirers, promote communication as they mostly require students to work in teams and encourage knowledge and reflection.

How does the course meet student needs via ATL?

The five approaches to learning (developing thinking skills, social skills, communication skills, self-management skills and research skills) along with the six approaches to teaching (teaching that is inquiry-based, conceptually-focused, contextualized, collaborative, differentiated and informed by assessment) encompass the key values and principles that underpin IB pedagogy.

Describe connections with CAS

Creativity, activity, service (CAS) is at the heart of the Diploma Programme. All courses have to be aligned and promote CAS activities. Students are engaged in various CAS activities that evolve maths as followed:

1. They create a mathematics scavenger hunt for younger students.
2. They assist other students in Math classes, usually younger students.
3. They create surveys and databases in order to resolve problems of the school community or area.

Describe connections with TOK

Theory of knowledge (TOK) is a course that is fundamentally about critical thinking and inquiry, the two attributes that are mostly required in mathematical knowledge. During the course Math students are given questions and topics that are expected to discuss during TOK classes and investigate. They are encouraged to investigate further and finally a TOK and Maths combined course is designed to promote the TOK and Maths connection.

Recommended resources

www.ibo.org

Khan Academy

TedEd

Instructor's name

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